Integration Technique - Standard Integrals

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Questions in past papers often come up combined with other topics.
Topic tags have been given for each question to enable you to know if you can do the question or whether you need to wait to cover the additional topic(s).

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Question 1

Qualification: AP Calculus AB

Areas: Differential Equations

Subtopics: Separation of Variables in Differential Equation, Particular Solution of Differential Equation, Integration Technique – Exponentials, Integration Technique – Standard Functions

Paper: Part B-Non-Calc / Series: 2000 / Difficulty: Medium / Question Number: 6

6. Consider the differential equation $\frac{dy}{dx} = \frac{3x^2}{e^{2y}}$.

(a) Find a solution $y = f(x)$ to the differential equation satisfying $f(0) = \frac{1}{2}$.

(b) Find the domain and range of the function f found in part (a).

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Question 2

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Integration, Differentiation

Subtopics: Points Of Inflection, Local or Relative Minima and Maxima, Integration Technique – Standard Functions, Differentiation Technique – Standard Functions

Paper: Part B-Non-Calc / Series: 2001 / Difficulty: Medium / Question Number: 5

5. A cubic polynomial function f is defined by

$$f(x) = 4x^3 + ax^2 + bx + k$$

where a, b, and k are constants. The function f has a local minimum at $x = -1$, and the graph of f has a point of inflection at $x = -2$.

(a) Find the values of a and b.

(b) If $\int_0^1 f(x)\,dx = 32$, what is the value of k ?

Question 3

Qualification: AP Calculus AB

Areas: Differentiation, Differential Equations

Subtopics: Implicit Differentiation, Separation of Variables in Differential Equation, Particular Solution of Differential Equation, Integration Technique – Standard Functions

Paper: Part B-Non-Calc / Series: 2001 / Difficulty: Somewhat Challenging / Question Number: 6

6. The function f is differentiable for all real numbers. The point $\left(3, \frac{1}{4}\right)$ is on the graph of $y = f(x)$, and the slope at each point (x, y) on the graph is given by $\frac{dy}{dx} = y^2(6 - 2x)$.

   (a) Find $\frac{d^2y}{dx^2}$ and evaluate it at the point $\left(3, \frac{1}{4}\right)$.

   (b) Find $y = f(x)$ by solving the differential equation $\frac{dy}{dx} = y^2(6 - 2x)$ with the initial condition $f(3) = \frac{1}{4}$.

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Question 4

Qualification: AP Calculus AB

Areas: Differential Equations, Applications of Differentiation

Subtopics: Tangents To Curves, Local or Relative Minima and Maxima, Particular Solution of Differential Equation, Separation of Variables in Differential Equation, Integration Technique – Standard Functions, Differentiation Technique - Quotient Rule

Paper: Part B-Non-Calc / Series: 2002-Form-B / Difficulty: Very Hard / Question Number: 5

5. Consider the differential equation $\frac{dy}{dx} = \frac{3 - x}{y}$.

(a) Let $y = f(x)$ be the particular solution to the given differential equation for $1 < x < 5$ such that the line $y = -2$ is tangent to the graph of f. Find the x-coordinate of the point of tangency, and determine whether f has a local maximum, local minimum, or neither at this point. Justify your answer.

(b) Let $y = g(x)$ be the particular solution to the given differential equation for $-2 < x < 8$, with the initial condition $g(6) = -4$. Find $y = g(x)$.

Question 5

Qualification: AP Calculus AB

Areas: Limits and Continuity, Integration, Applications of Integration

Subtopics: Continuities and Discontinuities, Calculating Limits Algebraically, Average Value of a Function, Properties of Integrals, Integration Technique – Standard Functions, Differentiability

Paper: Part B-Non-Calc / Series: 2003 / Difficulty: Very Hard / Question Number: 6

6. Let f be the function defined by

$$f(x) = \begin{cases} \sqrt{x+1} & \text{for } 0 \le x \le 3 \\ 5 - x & \text{for } 3 < x \le 5. \end{cases}$$

(a) Is f continuous at $x = 3$? Explain why or why not.

(b) Find the average value of $f(x)$ on the closed interval $0 \le x \le 5$.

(c) Suppose the function g is defined by

$$g(x) = \begin{cases} k\sqrt{x+1} & \text{for } 0 \le x \le 3 \\ mx + 2 & \text{for } 3 < x \le 5, \end{cases}$$

where k and m are constants. If g is differentiable at $x = 3$, what are the values of k and m?

Question 6

Qualification: AP Calculus AB

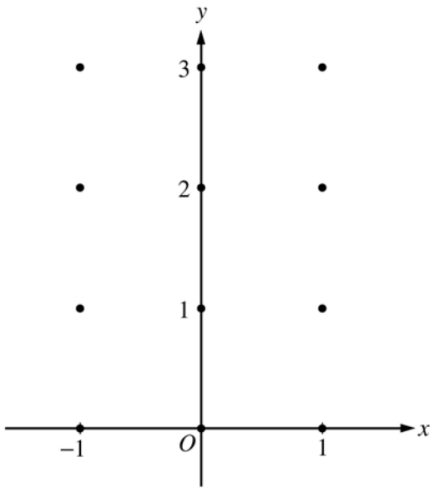
Areas: Differential Equations

Subtopics: Sketching Slope Field, Initial Conditions in Differential Equation, Separation of Variables in Differential Equation, Integration Technique – Standard Functions

Paper: Part B-Non-Calc / Series: 2004 / Difficulty: Medium / Question Number: 6

6. Consider the differential equation $\frac{dy}{dx} = x^2(y-1)$.

(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated. (**Note: Use the axes provided in the pink test booklet.**)



(b) While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the xy-plane. Describe all points in the xy-plane for which the slopes are positive.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 3$.

Question 7

Qualification: AP Calculus AB

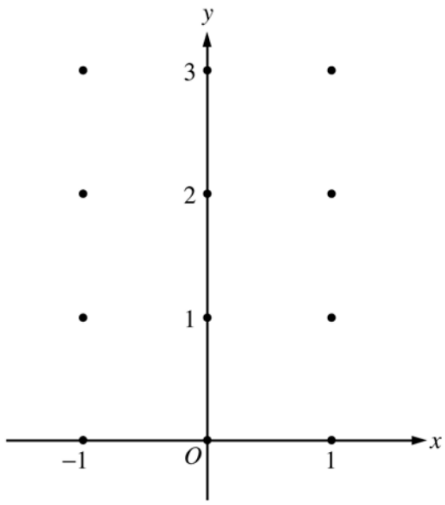
Areas: Differential Equations

Subtopics: Sketching Slope Field, Integration Technique - Harder Powers, Separation of Variables in Differential Equation, Particular Solution of Differential Equation, Integration Technique – Standard Functions

Paper: Part B-Non-Calc / Series: 2004-Form-B / Difficulty: Medium / Question Number: 5

5. Consider the differential equation $\frac{dy}{dx} = x^4(y-2)$.

(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated. **(Note: Use the axes provided in the test booklet.)**



(b) While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the xy-plane. Describe all points in the xy-plane for which the slopes are negative.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 0$.

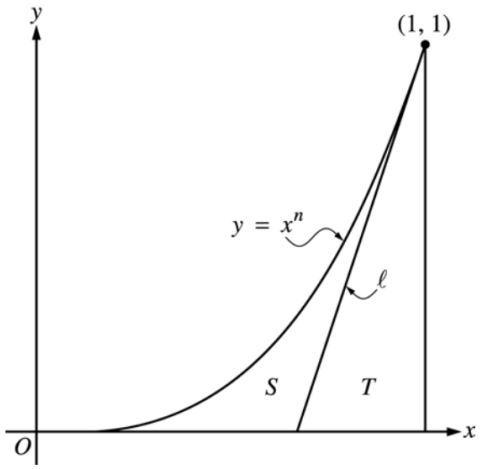
Question 8

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Integration, Applications of Integration

Subtopics: Integration Technique – Standard Functions, Integration Technique – Geometric Areas, Local or Relative Minima and Maxima, Differentiation Technique – Standard Functions

Paper: Part B-Non-Calc / Series: 2004-Form-B / Difficulty: Hard / Question Number: 6



6. Let ℓ be the line tangent to the graph of $y = x^n$ at the point $(1, 1)$, where $n > 1$, as shown above.

   (a) Find $\int_0^1 x^n \, dx$ in terms of n.

   (b) Let T be the triangular region bounded by ℓ, the x-axis, and the line $x = 1$. Show that the area of T is $\frac{1}{2n}$.

   (c) Let S be the region bounded by the graph of $y = x^n$, the line ℓ, and the x-axis. Express the area of S in terms of n and determine the value of n that maximizes the area of S.

Question 9

Qualification: AP Calculus AB

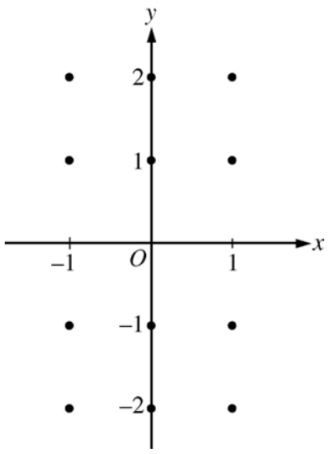
Areas: Differential Equations

Subtopics: Sketching Slope Field, Tangents To Curves, Separation of Variables in Differential Equation, Particular Solution of Differential Equation, Integration Technique – Standard Functions

Paper: Part B-Non-Calc / Series: 2005 / Difficulty: Medium / Question Number: 6

6. Consider the differential equation $\frac{dy}{dx} = -\frac{2x}{y}$.

(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated. (**Note: Use the axes provided in the pink test booklet.**)



(b) Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(1) = -1$. Write an equation for the line tangent to the graph of f at $(1, -1)$ and use it to approximate $f(1.1)$.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(1) = -1$.

Question 10

Qualification: AP Calculus AB

Areas: Differential Equations

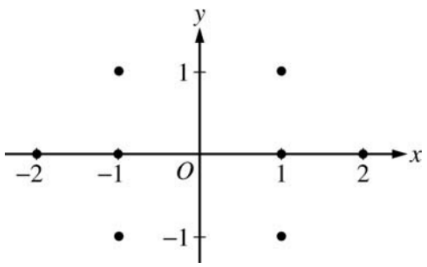
Subtopics: Initial Conditions in Differential Equation, Separation of Variables in Differential Equation, Sketching Slope Field, Integration Technique – Standard Functions

Paper: Part B-Non-Calc / Series: 2006 / Difficulty: Medium / Question Number: 5

5. Consider the differential equation $\frac{dy}{dx} = \frac{1+y}{x}$, where $x \neq 0$.

(a) On the axes provided, sketch a slope field for the given differential equation at the eight points indicated.

(Note: Use the axes provided in the pink exam booklet.)



(b) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(-1) = 1$ and state its domain.

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Question 11

Qualification: AP Calculus AB

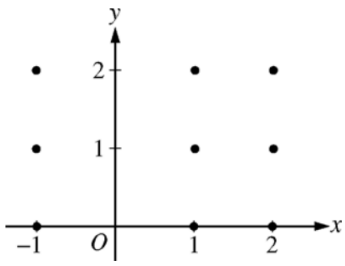
Areas: Differential Equations, Limits and Continuity

Subtopics: Sketching Slope Field, Particular Solution of Differential Equation, Initial Conditions in Differential Equation, Integration Technique – Standard Functions, Calculating Limits Algebraically

Paper: Part B-Non-Calc / Series: 2008 / Difficulty: Easy / Question Number: 5

5. Consider the differential equation $\frac{dy}{dx} = \frac{y-1}{x^2}$, where $x \neq 0$.

(a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.
(Note: Use the axes provided in the exam booklet.)



(b) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(2) = 0$.

(c) For the particular solution $y = f(x)$ described in part (b), find $\lim_{x \to \infty} f(x)$.

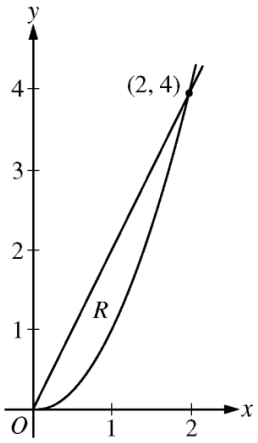
Question 12

Qualification: AP Calculus AB

Areas: Applications of Integration

Subtopics: Area Between Curves, Volume using Cross Sections, Integration Technique – Trigonometry, Integration Technique – Standard Functions

Paper: Part B-Non-Calc / Series: 2009 / Difficulty: Easy / Question Number: 4



4. Let R be the region in the first quadrant enclosed by the graphs of $y = 2x$ and $y = x^2$, as shown in the figure above.

   (a) Find the area of R.

   (b) The region R is the base of a solid. For this solid, at each x the cross section perpendicular to the x-axis has area $A(x) = \sin\left(\frac{\pi}{2}x\right)$. Find the volume of the solid.

   (c) Another solid has the same base R. For this solid, the cross sections perpendicular to the y-axis are squares. Write, but do not evaluate, an integral expression for the volume of the solid.

Question 13

Qualification: AP Calculus AB

Areas: Differential Equations, Applications of Differentiation

Subtopics: Tangents To Curves, Concavity, Particular Solution of Differential Equation, Initial Conditions in Differential Equation, Separation of Variables in Differential Equation, Integration Technique – Standard Functions

Paper: Part B-Non-Calc / Series: 2010 / Difficulty: Easy / Question Number: 6

6. Solutions to the differential equation $\frac{dy}{dx} = xy^3$ also satisfy $\frac{d^2y}{dx^2} = y^3\left(1 + 3x^2y^2\right)$. Let $y = f(x)$ be a particular solution to the differential equation $\frac{dy}{dx} = xy^3$ with $f(1) = 2$.

   (a) Write an equation for the line tangent to the graph of $y = f(x)$ at $x = 1$.

   (b) Use the tangent line equation from part (a) to approximate $f(1.1)$. Given that $f(x) > 0$ for $1 < x < 1.1$, is the approximation for $f(1.1)$ greater than or less than $f(1.1)$? Explain your reasoning.

   (c) Find the particular solution $y = f(x)$ with initial condition $f(1) = 2$.

Question 14

Qualification: AP Calculus AB

Areas: Differential Equations, Applications of Differentiation

Subtopics: Tangents To Curves, Concavity, Particular Solution of Differential Equation, Initial Conditions in Differential Equation, Separation of Variables in Differential Equation, Integration Technique – Standard Functions

Paper: Part B-Non-Calc / Series: 2011 / Difficulty: Medium / Question Number: 5

5. At the beginning of 2010, a landfill contained 1400 tons of solid waste. The increasing function W models the total amount of solid waste stored at the landfill. Planners estimate that W will satisfy the differential equation $\frac{dW}{dt} = \frac{1}{25}(W - 300)$ for the next 20 years. W is measured in tons, and t is measured in years from the start of 2010.

   (a) Use the line tangent to the graph of W at $t = 0$ to approximate the amount of solid waste that the landfill contains at the end of the first 3 months of 2010 (time $t = \frac{1}{4}$).

   (b) Find $\frac{d^2W}{dt^2}$ in terms of W. Use $\frac{d^2W}{dt^2}$ to determine whether your answer in part (a) is an underestimate or an overestimate of the amount of solid waste that the landfill contains at time $t = \frac{1}{4}$.

   (c) Find the particular solution $W = W(t)$ to the differential equation $\frac{dW}{dt} = \frac{1}{25}(W - 300)$ with initial condition $W(0) = 1400$.

Question 15

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Integration

Subtopics: Local or Relative Minima and Maxima, Concavity, Differentiation Technique – Standard Functions, Integration Technique – Standard Functions

Paper: Part B-Non-Calc / Series: 2011-Form-B / Difficulty: Medium / Question Number: 4

4. Consider a differentiable function f having domain all positive real numbers, and for which it is known that $f'(x) = (4 - x)x^{-3}$ for $x > 0$.

   (a) Find the x-coordinate of the critical point of f. Determine whether the point is a relative maximum, a relative minimum, or neither for the function f. Justify your answer.

   (b) Find all intervals on which the graph of f is concave down. Justify your answer.

   (c) Given that $f(1) = 2$, determine the function f.

Question 16

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Differential Equations

Subtopics: Derivative Graphs, Concavity, Separation of Variables in Differential Equation, Integration Technique – Standard Functions, Initial Conditions in Differential Equation, Particular Solution of Differential Equation

Paper: Part B-Non-Calc / Series: 2012 / Difficulty: Somewhat Challenging / Question Number: 5

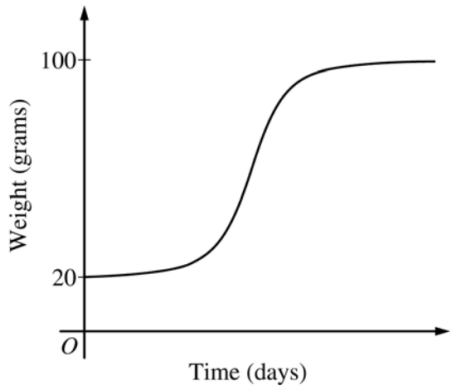
5. The rate at which a baby bird gains weight is proportional to the difference between its adult weight and its current weight. At time $t = 0$, when the bird is first weighed, its weight is 20 grams. If $B(t)$ is the weight of the bird, in grams, at time t days after it is first weighed, then

$$\frac{dB}{dt} = \frac{1}{5}(100 - B).$$

Let $y = B(t)$ be the solution to the differential equation above with initial condition $B(0) = 20$.

(a) Is the bird gaining weight faster when it weighs 40 grams or when it weighs 70 grams? Explain your reasoning.

(b) Find $\frac{d^2B}{dt^2}$ in terms of B. Use $\frac{d^2B}{dt^2}$ to explain why the graph of B cannot resemble the following graph.



(c) Use separation of variables to find $y = B(t)$, the particular solution to the differential equation with initial condition $B(0) = 20$.

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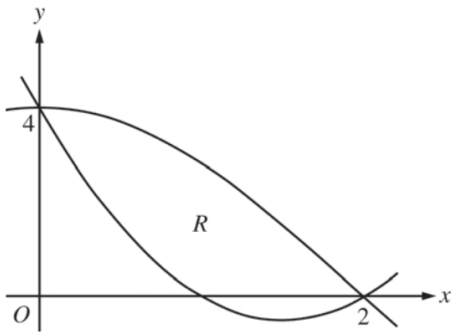
Question 17

Qualification: AP Calculus AB

Areas: Applications of Integration

Subtopics: Area Between Curves, Volume of Revolution – Washer Method, Volume using Cross Sections, Integration Technique – Trigonometry, Integration Technique – Standard Functions

Paper: Part B-Non-Calc / Series: 2013 / Difficulty: Medium / Question Number: 5



5. Let $f(x) = 2x^2 - 6x + 4$ and $g(x) = 4\cos\left(\frac{1}{4}\pi x\right)$. Let R be the region bounded by the graphs of f and g, as shown in the figure above.

   (a) Find the area of R.

   (b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 4$.

   (c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Write, but do not evaluate, an integral expression that gives the volume of the solid.

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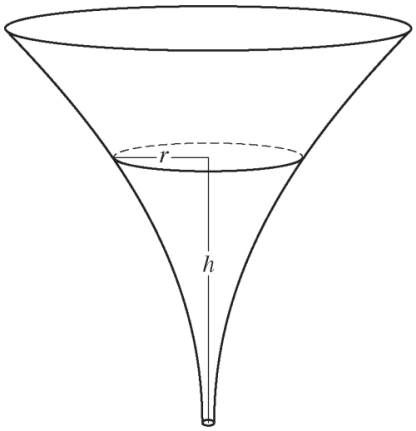
Question 18

Qualification: AP Calculus AB

Areas: Applications of Integration

Subtopics: Average Value of a Function, Volume of Revolution – Disc Method, Rates of Change (Instantaneous), Integration Technique – Standard Functions, Modelling Situations, Related Rates

Paper: Part B-Non-Calc / Series: 2016 / Difficulty: Medium / Question Number: 5



5. The inside of a funnel of height 10 inches has circular cross sections, as shown in the figure above. At height h, the radius of the funnel is given by $r = \frac{1}{20}\left(3 + h^2\right)$, where $0 \le h \le 10$. The units of r and h are inches.

   (a) Find the average value of the radius of the funnel.

   (b) Find the volume of the funnel.

   (c) The funnel contains liquid that is draining from the bottom. At the instant when the height of the liquid is $h = 3$ inches, the radius of the surface of the liquid is decreasing at a rate of $\frac{1}{5}$ inch per second. At this instant, what is the rate of change of the height of the liquid with respect to time?

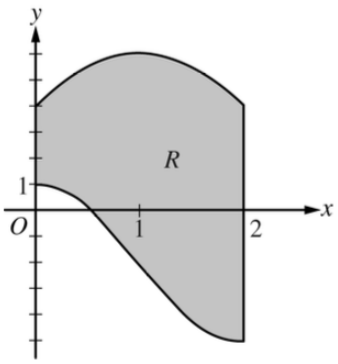
Question 19

Qualification: AP Calculus AB

Areas: Applications of Integration

Subtopics: Area Between Curves, Volume using Cross Sections, Volume of Revolution – Washer Method, Integration Technique – Trigonometry, Integration Technique – Standard Functions

Paper: Part B-Non-Calc / Series: 2019 / Difficulty: Medium / Question Number: 5



5. Let R be the region enclosed by the graphs of $g(x) = -2 + 3\cos\left(\frac{\pi}{2}x\right)$ and $h(x) = 6 - 2(x-1)^2$, the y-axis, and the vertical line $x = 2$, as shown in the figure above.

   (a) Find the area of R.

   (b) Region R is the base of a solid. For the solid, at each x the cross section perpendicular to the x-axis has area $A(x) = \frac{1}{x+3}$. Find the volume of the solid.

   (c) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 6$.

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